

# 2IMP25 Software Evolution

## Software metrics

Alexander Serebrenik



**TU** / **e**

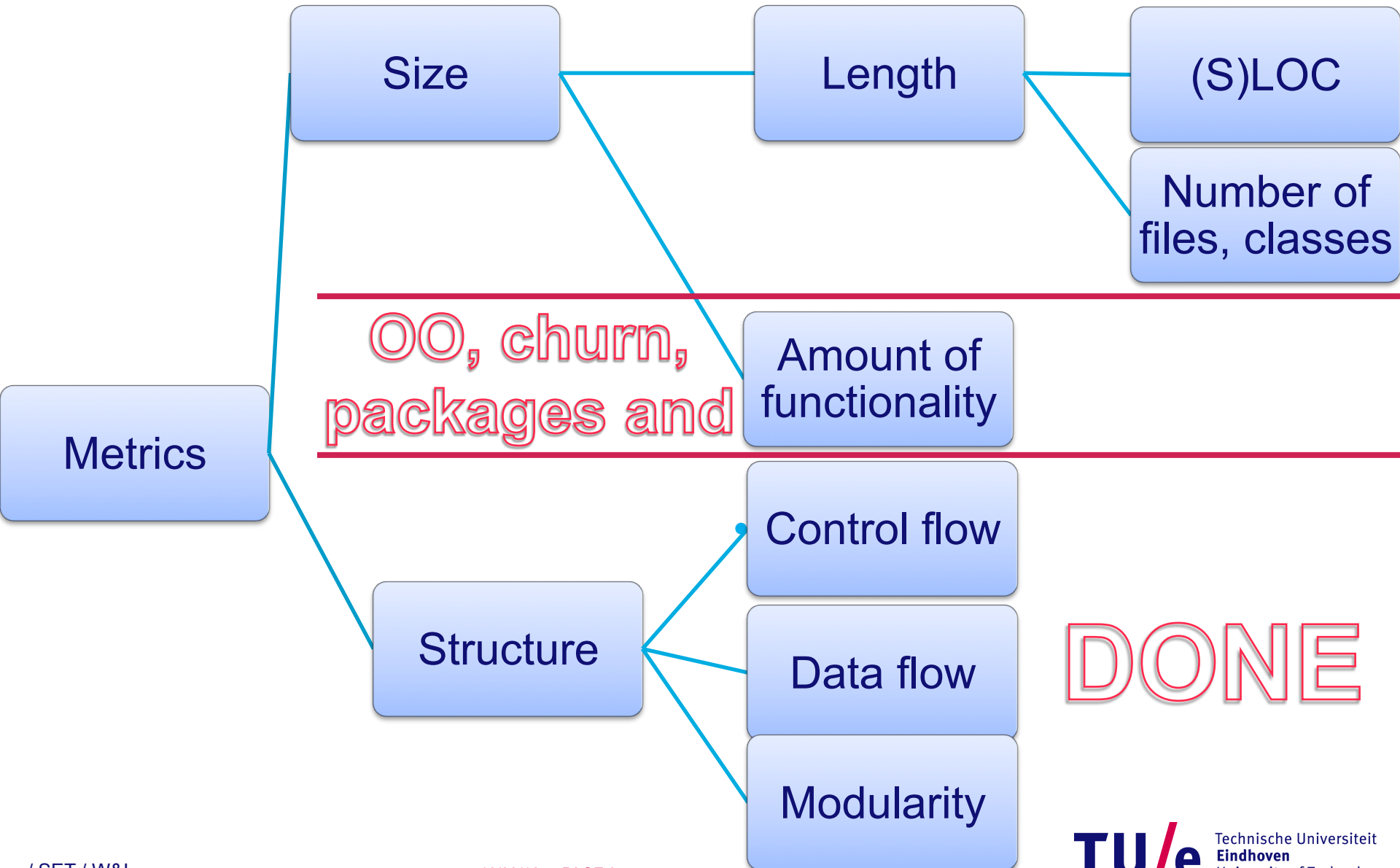
Technische Universiteit  
**Eindhoven**  
University of Technology

Where innovation starts

# Assesement

- **Assignment 2: Thank you for submitting!**
  - **Median 7, mean 6.56**
  - **First feedback: 3 5 4 5 1**
    - **Like:** topic, tool chain, compilers, visual aspect
    - **Don't like:**
      - Building tools vs using tools
      - Rascal learning curve
      - Spelling
- **Assignment 3: March 30, 23:59**

# Where are we now?



# From imperative to OO

- All metrics so far were designed for imperative languages
  - Applicable for OO
    - On the method level
    - Also
      - Number of files → number of classes/packages
      - Fan-in → afferent coupling ( $C_a$ )
      - Fan-out → efferent coupling ( $C_e$ )
  - But do not reflect OO-specific complexity
    - Inheritance, class fields, abstractness, ...
- Popular metric sets
  - Chidamber and Kemerer, Li and Henry, Lorenz and Kidd, Abreu, Martin

# Chidamber and Kemerer

- **WMC – weighted methods per class**
  - Sum of metrics(m) for all methods m in class C
- **DIT – depth of inheritance tree**
  - java.lang.Object? Libraries?
- **NOC – number of children**
  - Direct descendents
- **CBO – coupling between object classes**
  - A is coupled to B if A uses methods/fields of B
  - $CBO(A) = | \{B | A \text{ is coupled to } B\} |$
- **RFC - #methods that can be executed in response to a message being received by an object of that class.**

# Modularity metrics: LCOM

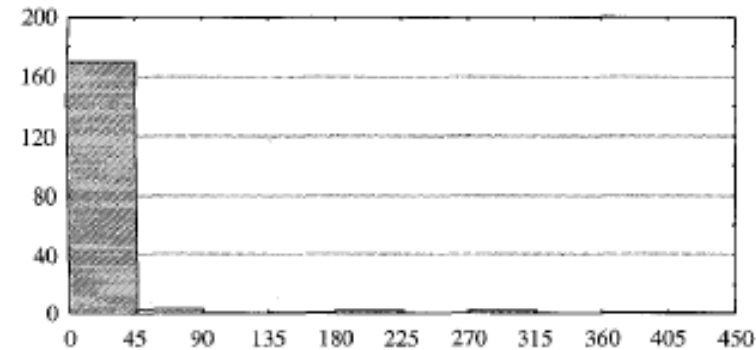
- **LCOM – lack of cohesion of methods**

- **Chidamber Kemerer:**

$$LCOM(C) = \begin{cases} P - Q & \text{if } P > Q \\ 0 & \text{otherwise} \end{cases}$$

**where**

- $P$  = #pairs of distinct methods in  $C$  that do not share instance variables
- $Q$  = #pairs of distinct methods in  $C$  that share instance variables
- Do you remember what is an **instance variable**?



**[BBM] 180 classes**

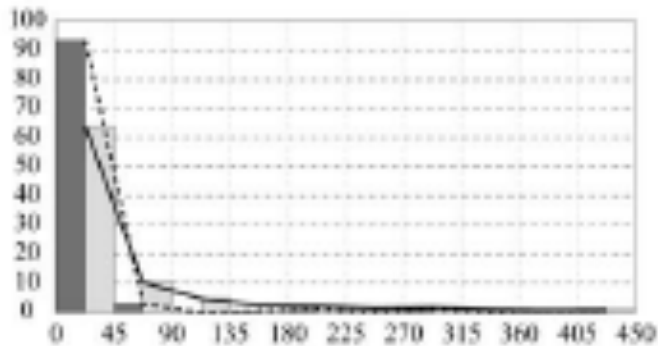
**Discriminative ability is insufficient**

**What about methods that use get/set instead of direct access?**

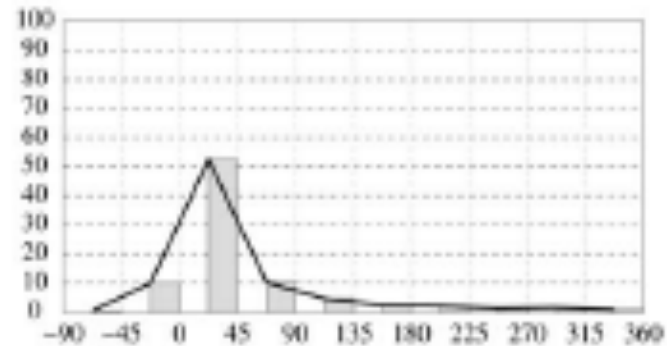
# First solution: LCOMN

- Defined similarly to LCOM but allows negative values

$$LCOMN(C) = P - Q$$



**LCOM**



**LCOMN**

- $m$  – number of methods
- $v$  – number of variables (attrs)
- $m(V_i)$  - #methods that access  $V_i$

$$\frac{\left( \frac{1}{v} \sum_{i=1}^v m(V_i) \right) - m}{1 - m}$$

- **Cohesion is maximal: all methods access all variables**  
 $m(V_i) = m$  and  $LCOM = 0$
- **No cohesion: every method accesses a unique variable**  
 $m(V_i) = 1$  and  $LCOM = 1$
- **Can LCOM exceed 1?**



# LCOM > 1?

- If some variables are not accessed at all, then

$$m(V_i) = 0$$

and if no variables are accessed

$$\frac{\left(\frac{1}{v} \sum_{i=1}^v m(V_i)\right) - m}{1 - m} = \frac{-m}{1 - m} = 1 + \frac{1}{m - 1}$$

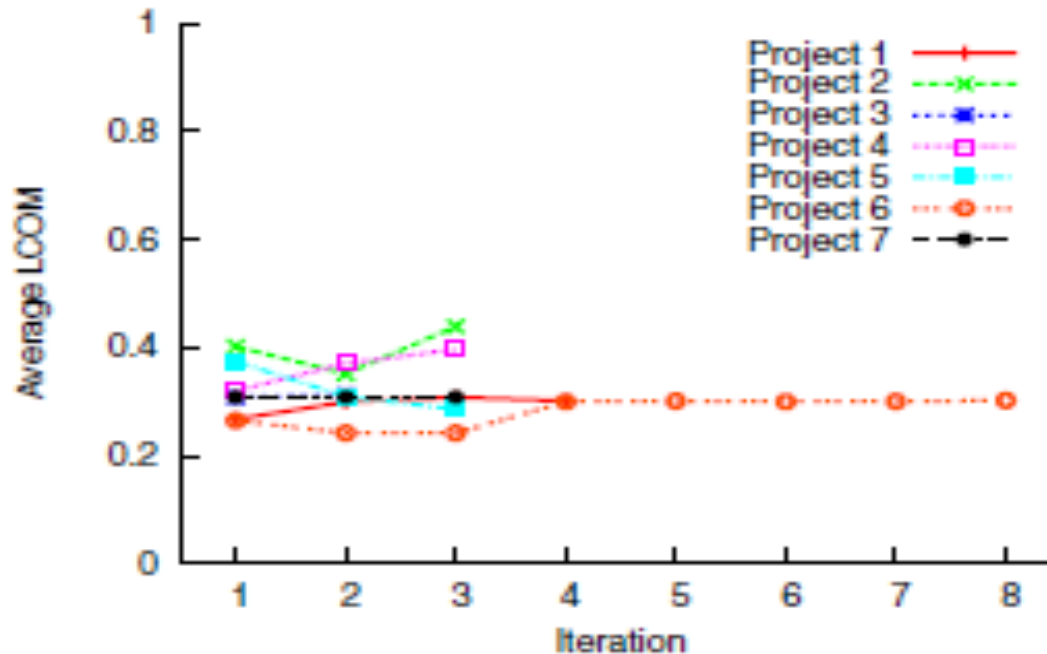
Hence

**LCOM is undefined for  $m = 1$**

**LCOM  $\leq 2$**

# Evolution of LCOM [Henderson-Sellers et al.]

Sato, Goldman,  
Kon 2007



- **Project 6 (commercial human resource system) suggests stabilization, but no similar conclusion can be made for other projects**

# Shortcomings of LCOM [Henderson-Sellers ]

Due to [Fernández, Peña 2006]

A	Variables			
M e t h o d s	Dark	Dark	Light	Light
	Dark	Dark	Light	Light
	Light	Light	Dark	Dark
	Light	Light	Dark	Dark

B	Variables			
M e t h o d s	Dark	Dark	Light	Light
	Light	Dark	Dark	Light
	Light	Light	Dark	Dark
	Dark	Light	Light	Dark

C	Variables			
M e t h o d s	Dark	Light	Light	Light
	Dark	Dark	Dark	Light
	Light	Dark	Dark	Dark
	Light	Light	Light	Dark

- **Method-variable diagrams: dark spot = access**
- **LCOM(A) ? LCOM(B) ? LCOM(C) ?**

$$\frac{\left( \frac{1}{v} \sum_{i=1}^v m(V_i) \right) - m}{1 - m}$$

# Shortcomings of LCOM [Henderson-Sellers ]

Due to [Fernández, Peña 2006]

A	Variables			
M e t h o d s	■	■	□	□
	■	■	□	□
	□	□	■	■
	□	□	■	■

B	Variables			
M e t h o d s	■	■	□	□
	□	■	■	□
	□	□	■	■
	■	□	□	■

C	Variables			
M e t h o d s	■	□	□	□
	■	■	■	□
	□	■	■	■
	□	□	□	■

- All LCOM values are the same: 0.67
  - $m=4$ ,  $m(V_i) = 2$  for all  $i$
- A seems to be less cohesive than B and C!

$$\frac{\left( \frac{1}{v} \sum_{i=1}^v m(V_i) \right) - m}{1 - m}$$

# Alternative [Hitz, Montazeri 1995]

- **LCOM as the number of strongly connected components in the following graph**
  - **Vertices: methods**
    - **except for getters/setters**
  - **Edge between a and b, if**
    - **a and b access the same variable**
- **LCOM values**
  - **0, no methods**
  - **1, cohesive component**
  - **2 or more, lack of cohesion**

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**Question: LCOM?**

<b>A</b>	<b>Variables</b>			
<b>M et ho ds</b>	■	■	□	□
	■	■	□	□
	□	□	■	■
	□	□	■	■

<b>B</b>	<b>Variables</b>			
<b>M et ho ds</b>	■	■	□	□
	□	■	■	□
	□	□	■	■
	■	□	□	■

# Experimental evaluation of LCOM variants

Cox, Etkorn and Hughes 2006	Correlation with expert assessment	
	Group 1	Group 2
Chidamber Kemerer	-0.43 (p = 0.12)	-0.57 (p = 0.08)
Henderson-Sellers	-0.44 (p = 0.12)	-0.46 (p = 0.18)
Hitz, Montazeri	-0.47 (p = 0.06)	-0.53 (p = 0.08)

Etkorn, Gholston, Fortune, Stein, Utley, Farrington, Cox	Correlation with expert assessment	
	Group 1	Group 2
Chidamber Kemerer	-0.46 (rating 5/8)	-0.73 (rating 1.5/8)
Henderson-Sellers	-0.44 (rating 7/8)	-0.45 (rating 7/8)
Hitz, Montazeri	-0.51 (rating 2/8)	-0.54 (rating 5/8)

# LCC and TCC [Bieman, Kang 1994]

- Recall: LCOM HM “a and b access the same variable”
- What if a calls a', b calls b', and a' and b' access the same variable?
- Metrics
  - **NDP** – number of pairs of methods directly accessing the same variable
  - **NIP** – number of pairs of methods directly or indirectly accessing the same variable
  - **NP** – number of pairs of methods:  $n(n-1)/2$
- Tight class cohesion **TCC** = **NDP/NP**
- Loose class cohesion **LCC** = **NIP/NP**
- **NB: Constructors and destructors are excluded**



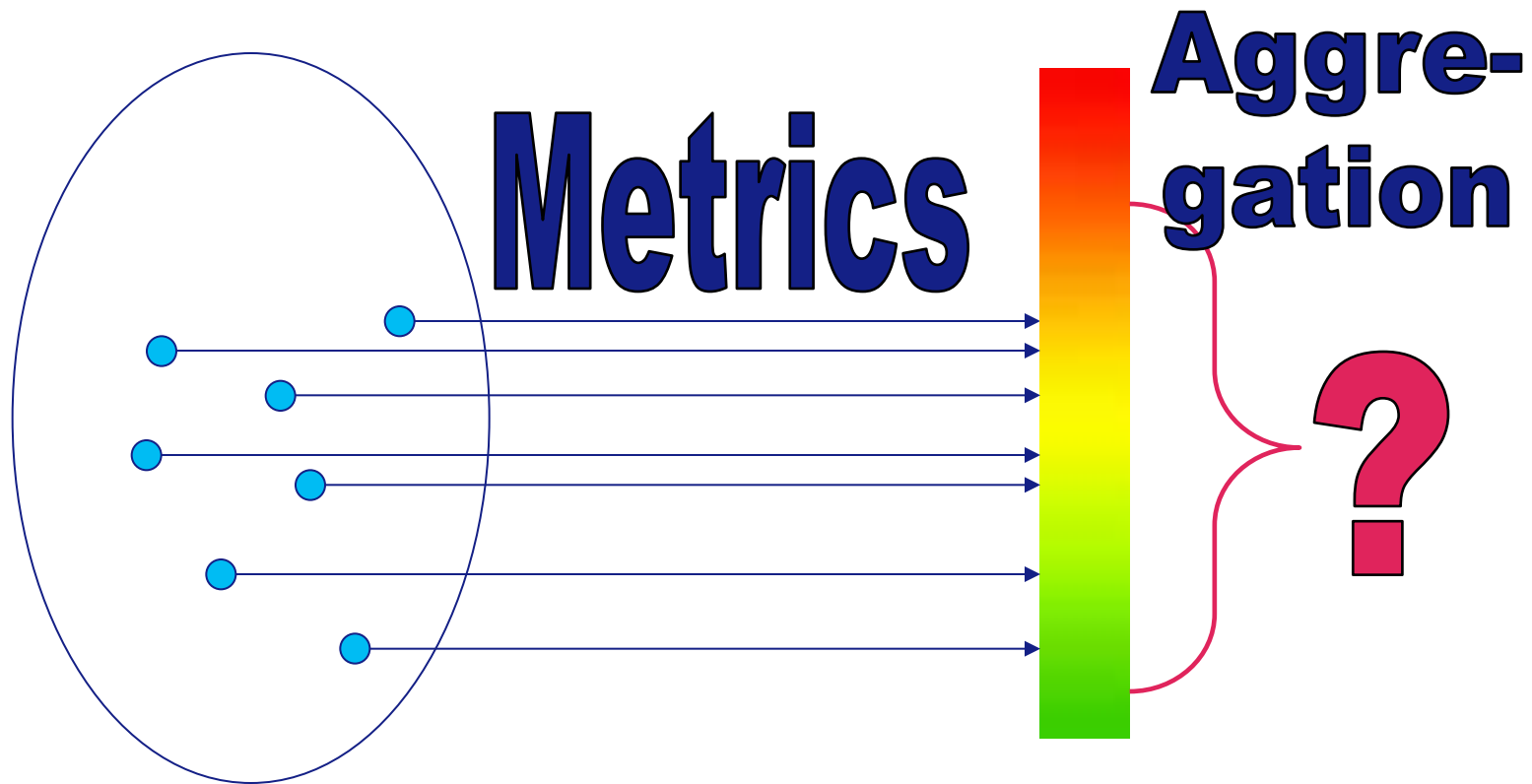
# Experimental evaluation of LCC/TCC

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Hitz, Montazeri	-0.51 (rating 2/8)	-0.54 (rating 5/8)
TCC	-0.22 (rating 8/8)	-0.057 (rating 8/8)
LCC	-0.54 (rating 1/8)	-0.73 (rating 1.5/8)

# Metrics so far...

<b>Level</b>	<b>Metrics</b>
<b>Method</b>	<b>LOC, McCabe</b>
<b>Class</b>	<b>WMC, NOC, DIT, LCOM (and variants), LCC/TCC</b>
<b>Packages</b>	<b>???</b>

# Metrics for higher-level objects as aggregation of metrics for low-level objects



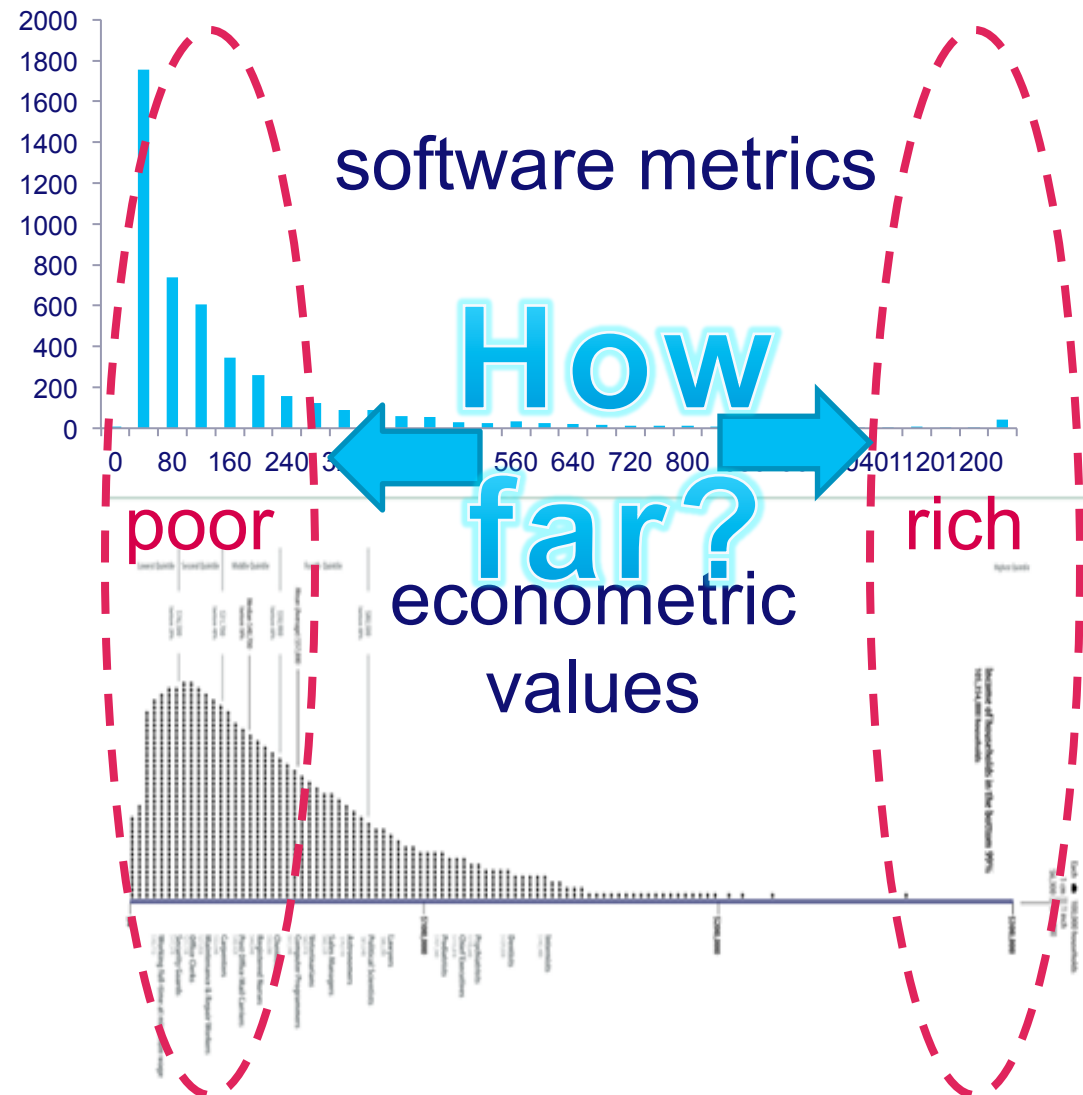
# Aggregation techniques

- **Metrics-independent**
  - **Applicable for any metrics to be aggregated**
    - **Traditional: mean, median...**
      - **“By no means”**
    - **Econometric: inequality indices**
- **Metrics-dependent**
  - **Produce more precise results**
  - **BUT: need to be redone for any new metrics**
  - **Based on fitting probability distributions**

# Metrics independent: Coefficient of variation

- **Coefficient of variation:  $C = \sigma/\mu$** 
  - **Allows to compare distributions with different means**
  - **Sometimes used to assess stability of the metrics**
    - **Metrics is stable for  $C < 0.3$**
    - **Unreliable for small samples**
    - **Evolution should be studied...**

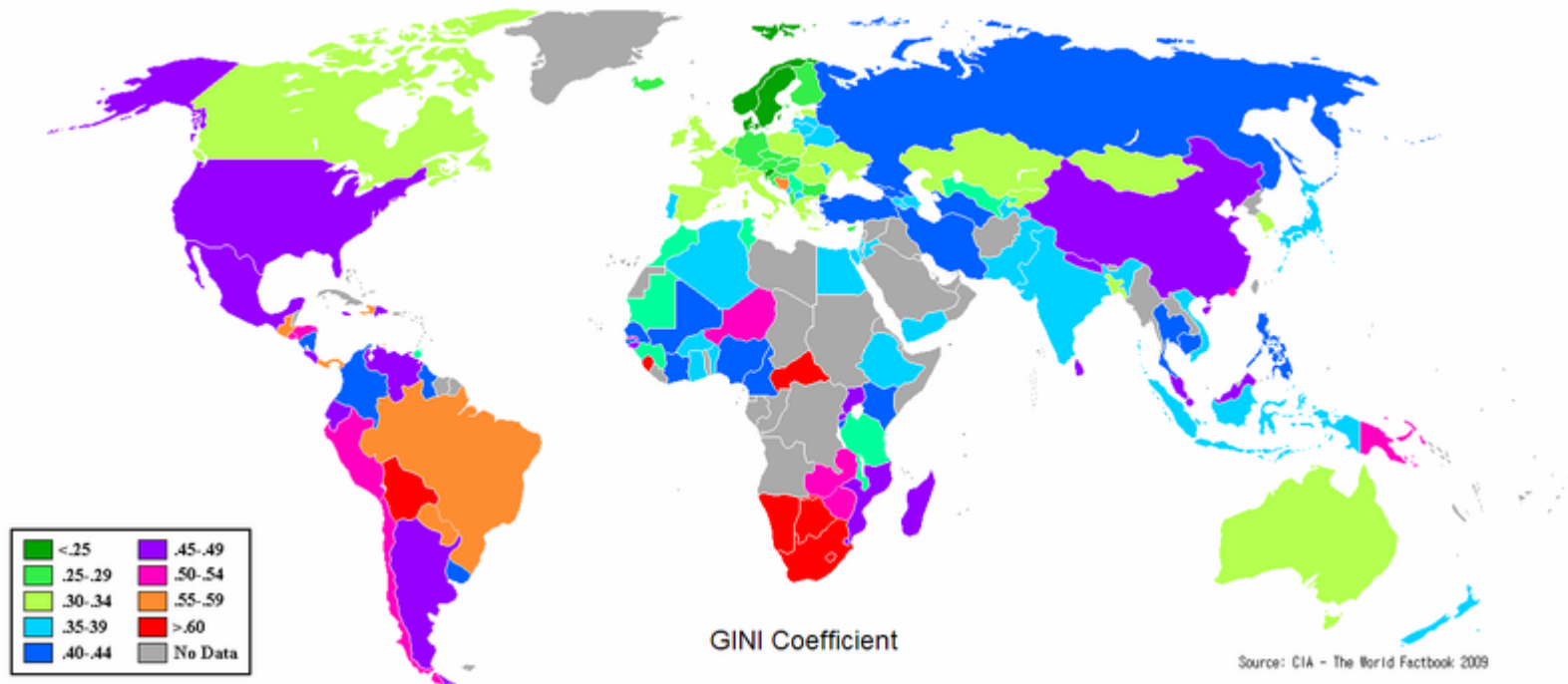
# Metrics are like money



- prog. lang.
- domain
- ...

- region
- education
- gender
- ...

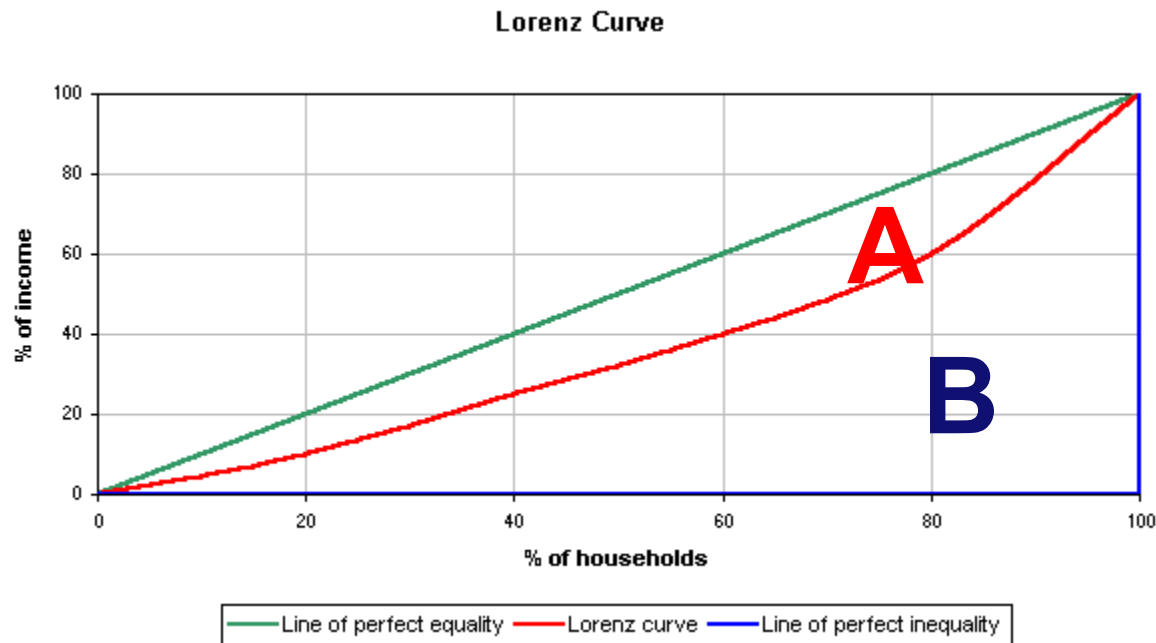
# Popular technique: Gini coefficient



- Gini coefficient measure of economic inequality
- Ranges on  $[0; 1 - 1/n]$
- High values indicate high inequality

# Gini coefficient: Formally

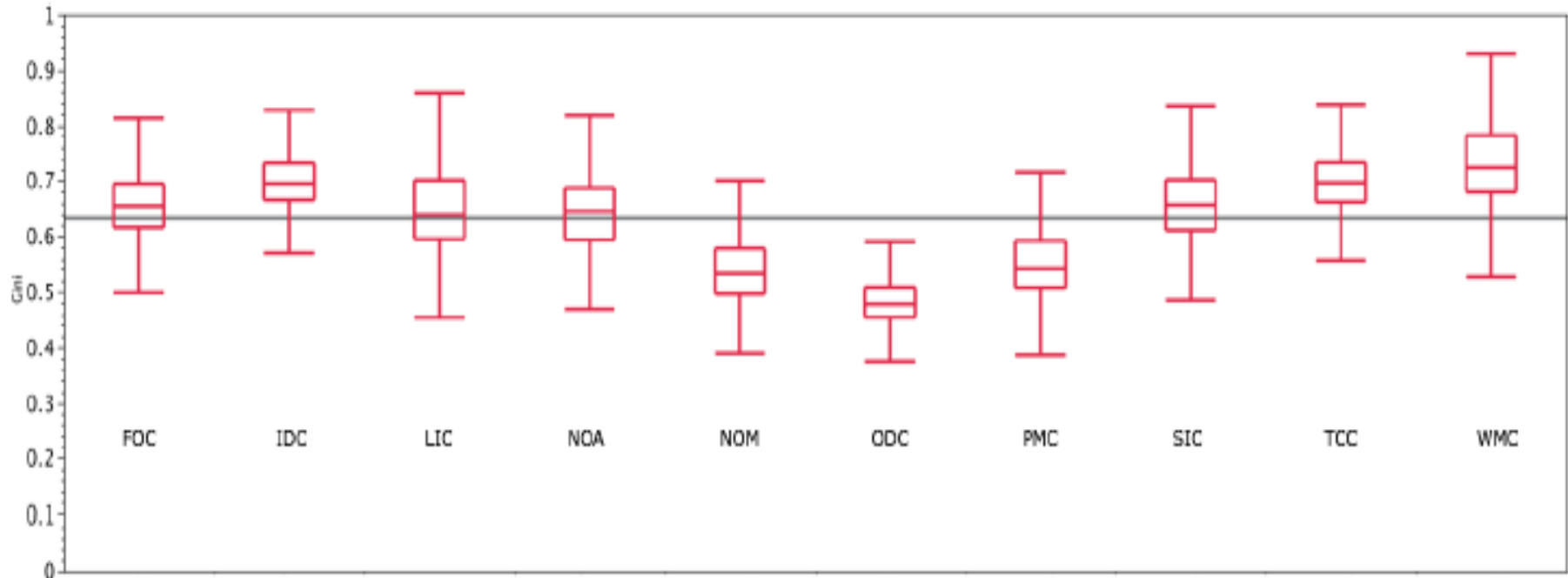
- **Lorenz curve:**
  - % of income shared by the lower % of the population



- **Gini =  $A/(A+B)$**
- **Since  $A+B = 0.5$**   
**Gini =  $2A$**



# Gini and software metrics [Vasa et al. 2009]

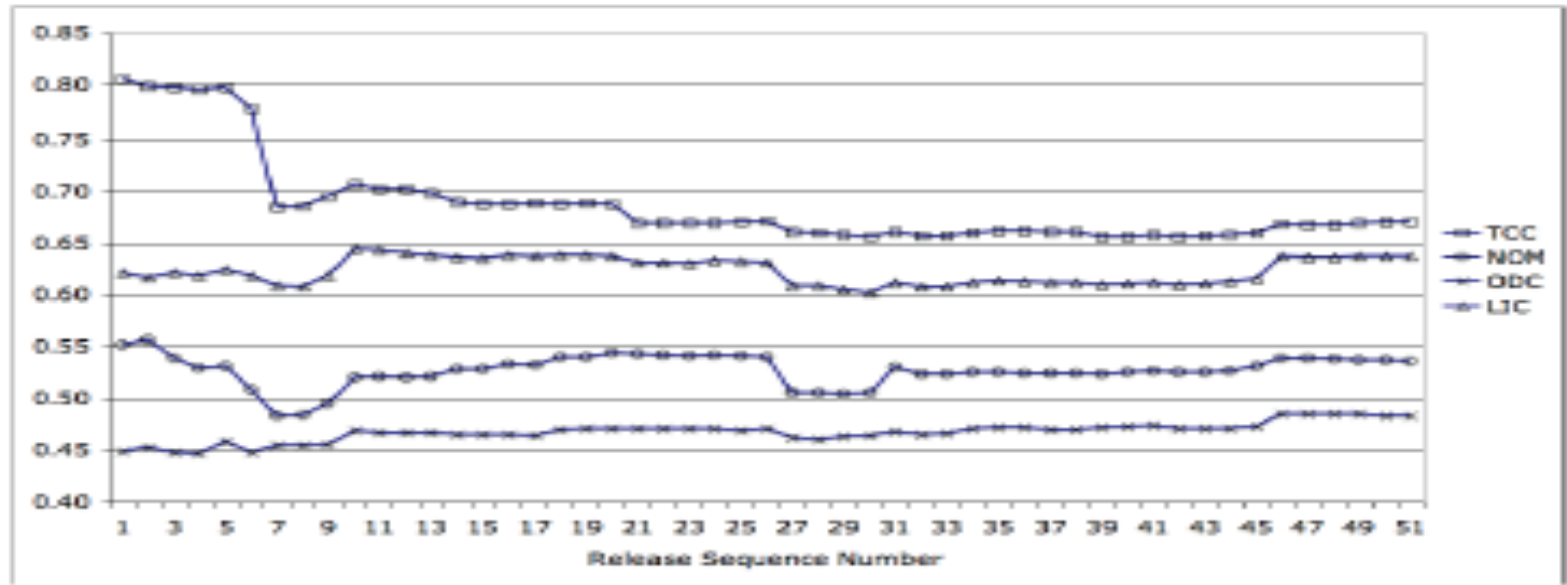


- For most of the metrics on the benchmark systems:  $0.45 \leq \text{Gini} \leq 0.75$
- Higher Gini/WMC: presence of **generated code** or code, structured in a way similar to the generated code (parsers)

# Gini and metrics: Exceptions

<b>System</b>	<b>Metrics</b>	<b>Increase</b>		<b>Explanation</b>
JabRef	WMC	0.75	0.91	Machine generated parser introduced
Checkstyle	Fan-in (classes)	0.44	0.80	Plug-in based architecture introduced.
Jasper-Reports	#Public methods	0.58	0.69	Introduction of a set of new base classes.
WebWork	Fan-out	0.51	0.62	A large utility class and multiple cases of copy-and paste introduced.

# Gini and evolution: Spring



- **Rather stable: programmers accumulate competence and tend to solve similar problems by similar means**
- **Similar for other econometric techniques: Theil, Hoover, Atkinson, ...**

# Aggregation techniques

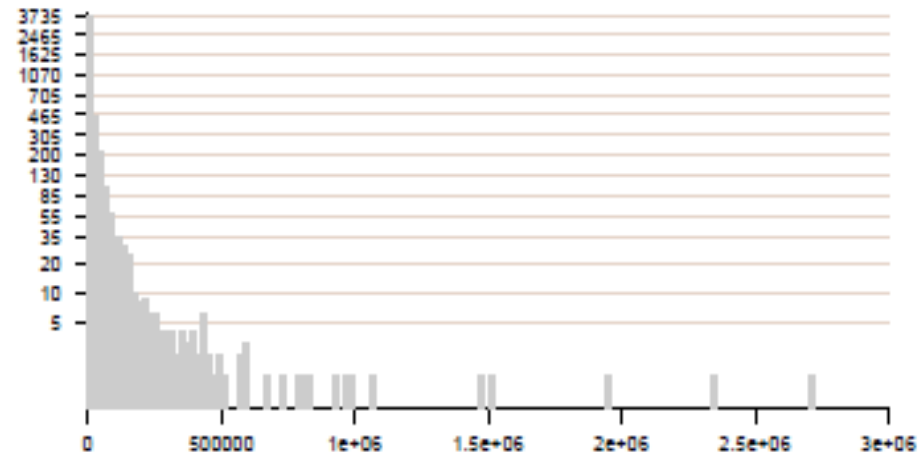
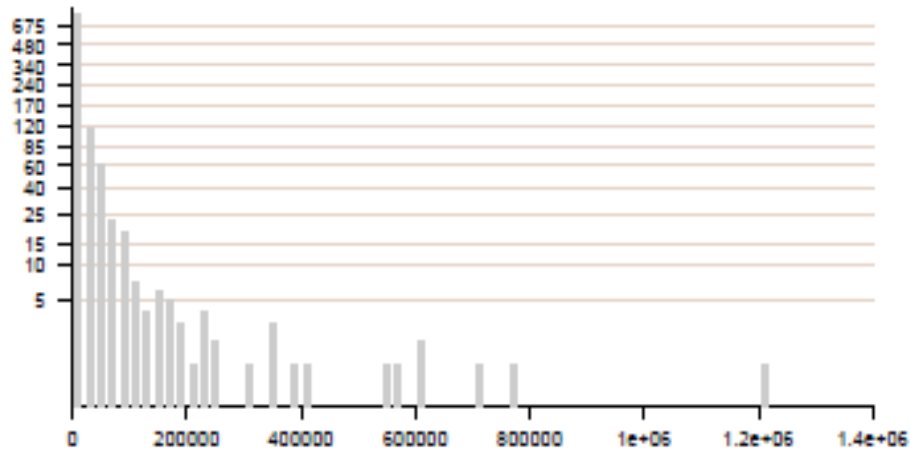
- **Metrics-independent**
  - **Applicable for any metrics to be aggregated**
  - **Are the results also metrics-independent?**
  - **Based on econometrics**
- **Metrics-dependent**
  - **Produces more precise results**
  - **BUT: needs to be redone for any new metrics**
  - **Based on fitting probability distributions**

# Metrics-dependent aggregation: Statistical fitting

1. Collect the metrics values for the lower-level elements
2. Present a **histogram**
3. Fit a (theoretical) probability distribution to describe the sample distribution
  - a) Select a **family** of theoretical distributions
  - b) Fit the **parameters** of the probability distribution
  - c) Assess the **goodness of fit**
4. If a theoretical distribution can be fitted, use the fitted parameters as the **aggregated value**

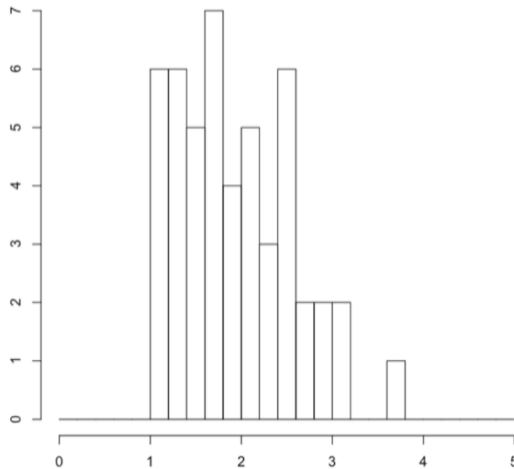
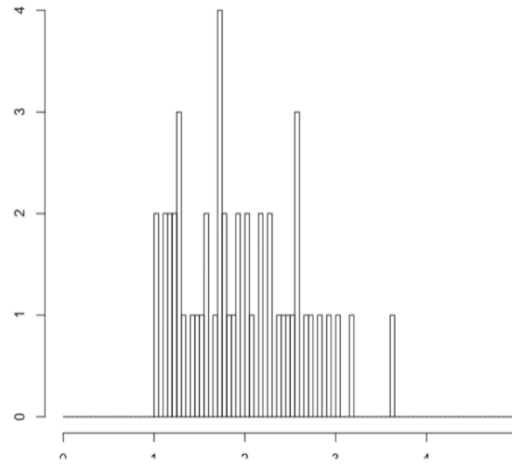
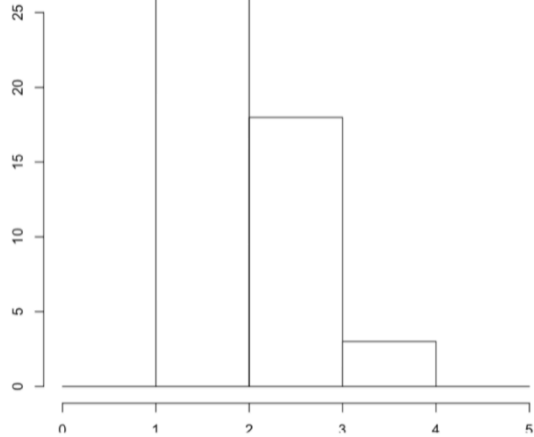
# Step 1: Histograms

- We have seen quite a number of them already!



**Robles et al. 2006: LOC in Debian 2.0 (left) and 3.0 (right)**

# Data: 50 birth weights of children with a severe idiopathic respiratory syndrome



- The same data leads to four different “distributions”
- What can affect the way histogram looks like?
  - Bin width
  - Position of the bin’s edges

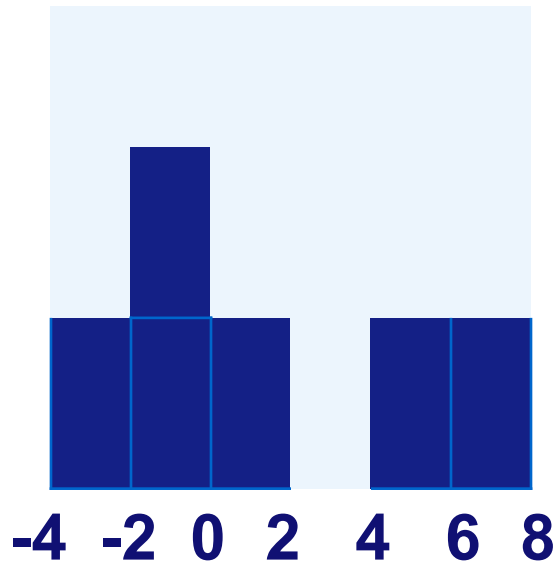
# Kernel density estimators

- **Advantages**
  - **Statistically more sound (no dependency on the end-points of the bins)**
  - **Produces smooth curves**
- **Disadvantages**
  - **Statistically more complex**
  - **Parameter tuning might be a challenge**



# Kernel density estimates: Intuition

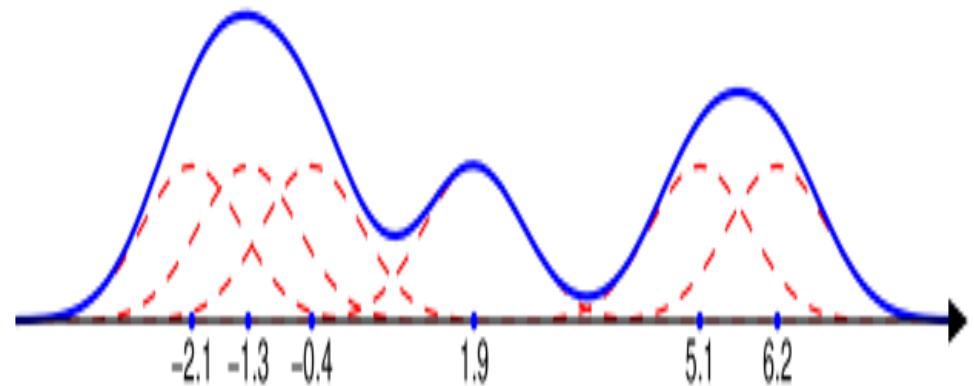
- Data: -2.1, -1.3, -0.4, 1.9, 5.1, 6.2



Histogram: every value is a rectangle.

Shape is a “sum” of the rectangles.

What if each value will be a “bump” that can be added together to create a smooth curve?



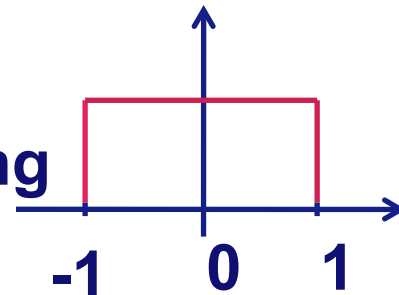
# Kernel density estimation: Formally

$$f(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x - x_i}{h}\right)$$

## Where

- $n$  – number of observations
- $h$  – a smoothing parameter, the “bandwidth”
- $K$  – a weighting function, the “kernel”

Histogram can be obtained using

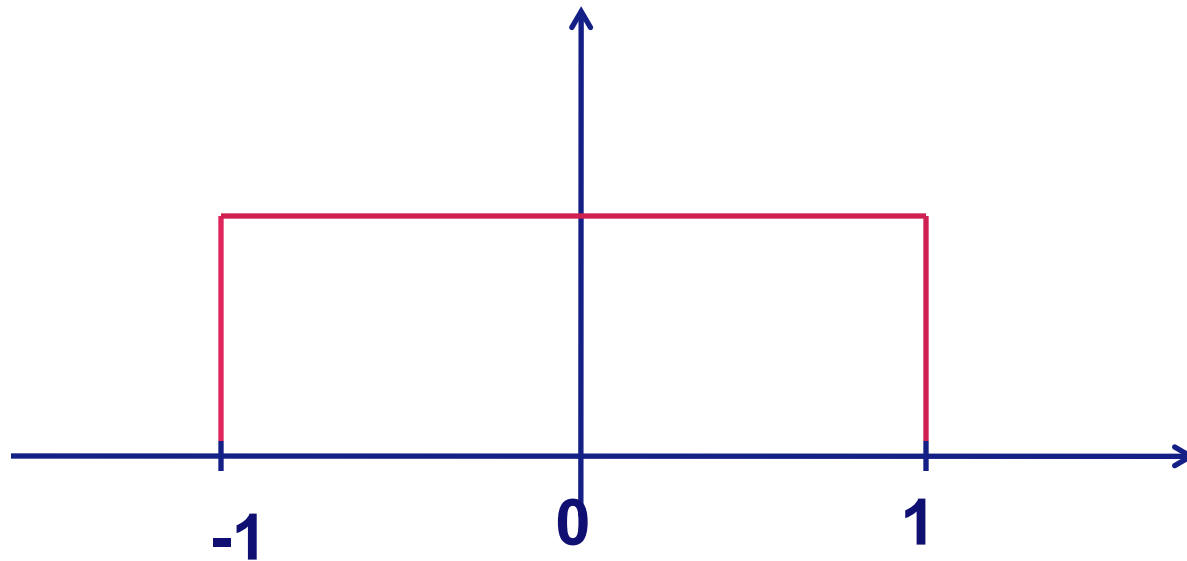


Once  $K$  is chosen one can determine the optimal  $h$ .

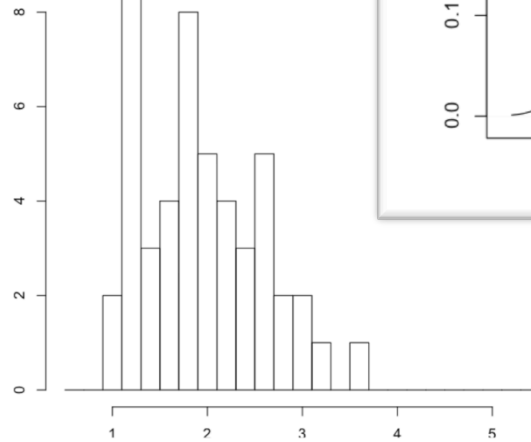
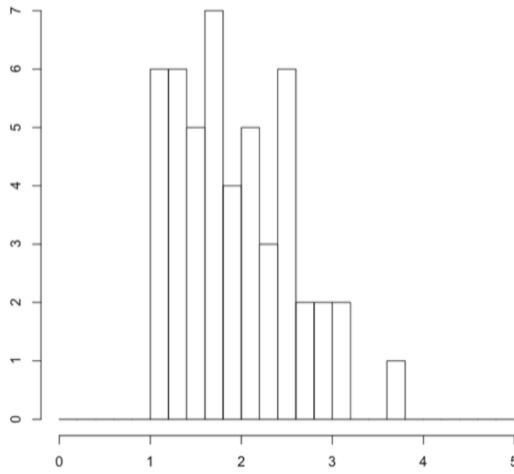
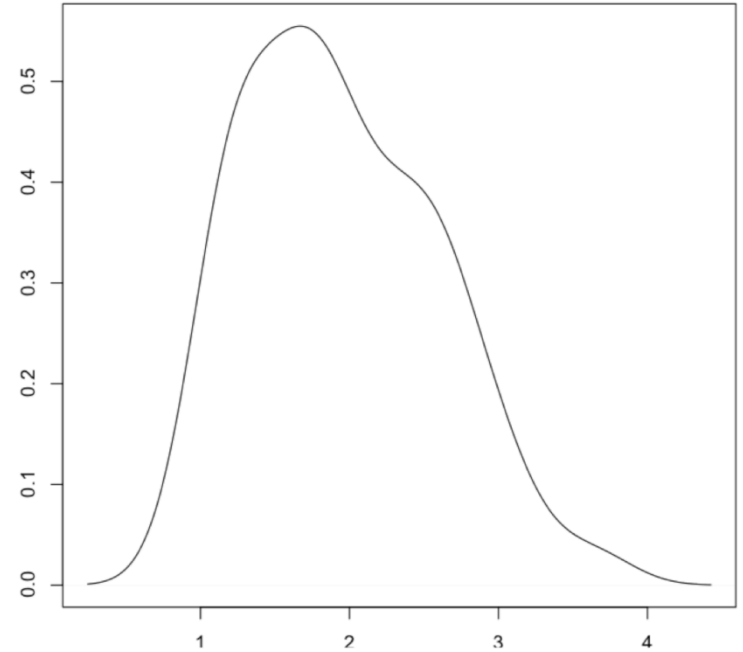
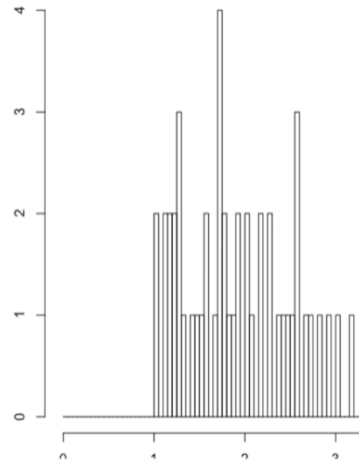
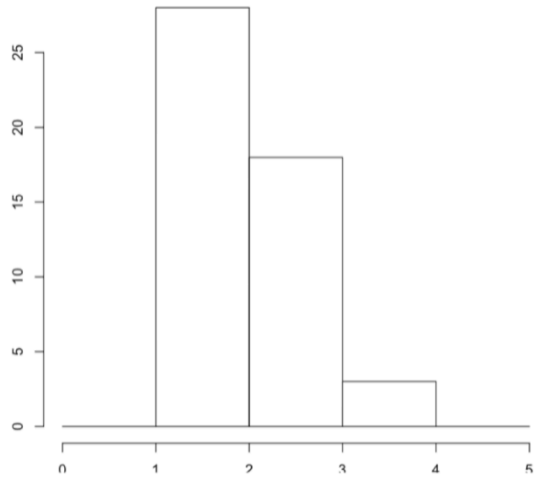
# Histogram as a kern density estimate

- $h$  is the bin width

- $x$  and  $x_i$  are in the same bin if  $-1 \leq \frac{x - x_i}{h} \leq 1$

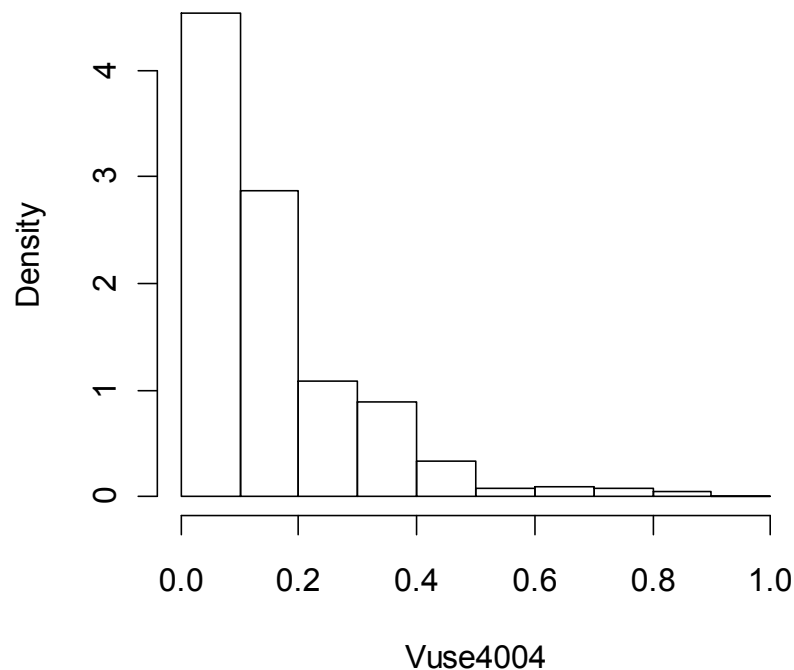


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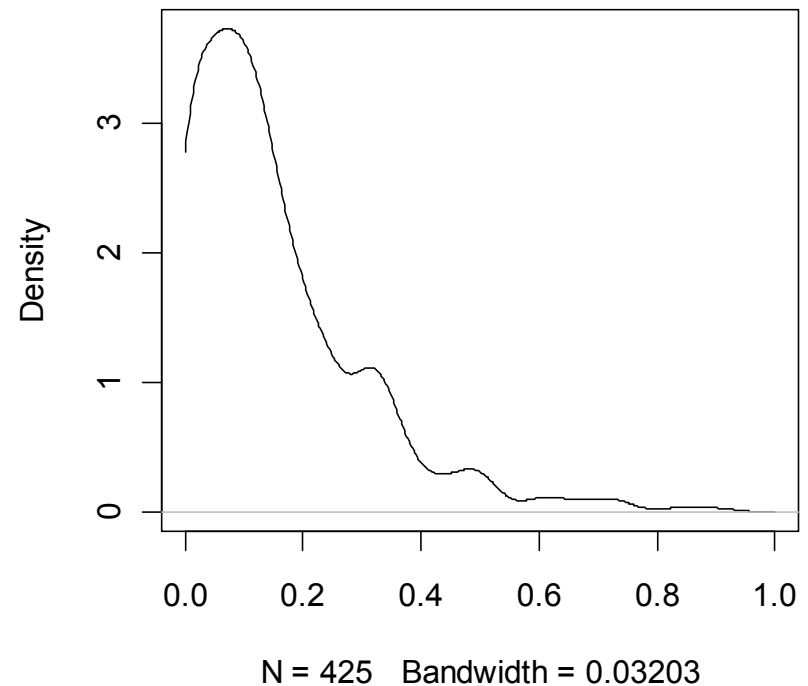


# Histogram vs. Kernel density estimate

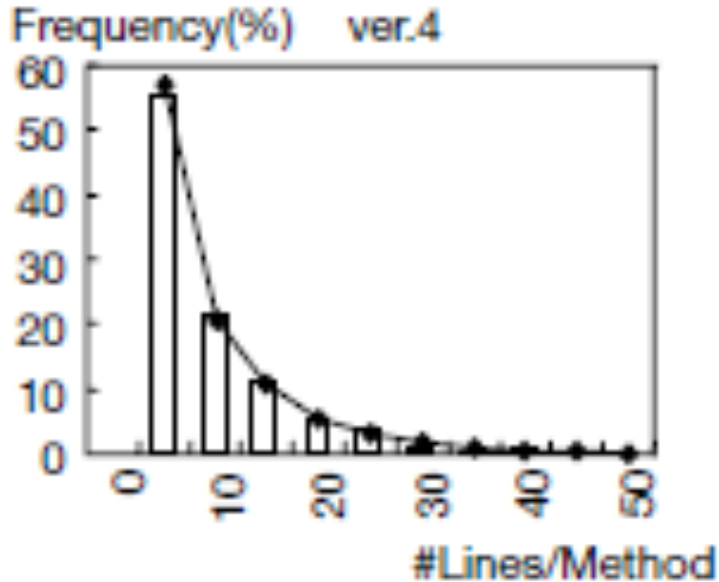
Histogram



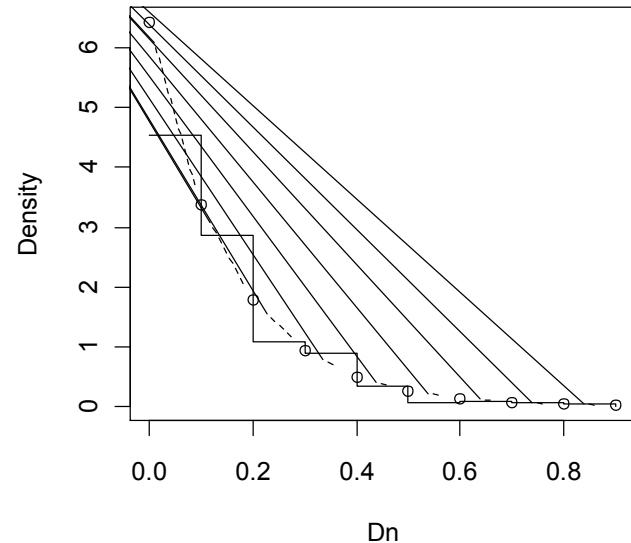
Kernel density estimate



# Step 2: fitting a distribution



**Tamai, Nakatani.**  
**Negative binomial**  
**distribution**



**S, Roubtsov, vd Brand**  
**“Exponential” distribution**

- Family of distributions is chosen based on shape
- If the parameters fitting is not **good enough** try a different one!

# Sometimes well-known distributions do not really seem to match

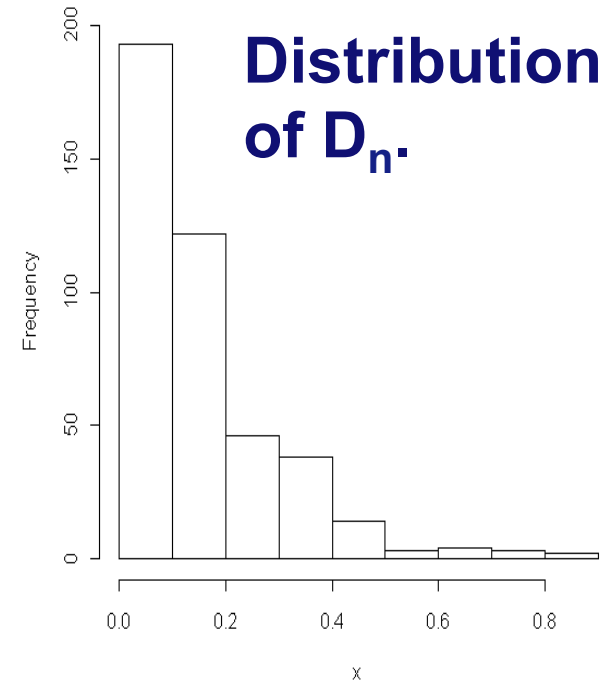
- **Exponential distribution:**

$$f(x) = \lambda e^{-\lambda x}$$

- **However, support is  $[0;1]$  rather than  $[0;\infty)$ !**

- **Since**  $\int_0^1 f(x) dx = 1 - e^{-\lambda}$

- **we normalize:** 
$$g(x) = \frac{f(x)}{\int_0^1 f(x) dx}$$



- **And use *max-likelihood fitting* to find  $\lambda$**

## Step 3c. Goodness of fit: Pearson $\chi^2$ test

- The test statistic

where

$$X^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$$

- O – observed frequency of the result i
- E – expected frequency of the result i
- Compare  $X^2$  with the theoretical  $\chi^2$  distribution for the given number of degrees of freedom
  - Degrees of freedom = number of observations – number of fitted parameters
  - Comparison is done based on table values
  - If the  $X^2 < \text{table value}$  – the fit is good



# Recapitulation: Statistical fitting

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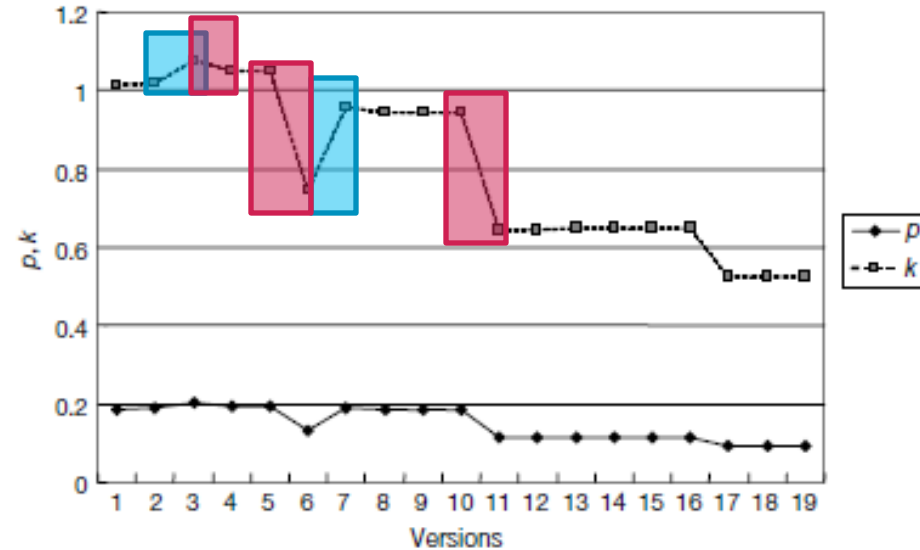
# What about the evolution of the aggregated values?

- **Geometry library: Jun, subsystem “Geometry”**
- **Tamai, Nakatani: Negative binomial distribution**

$$f(x) = \binom{x-1}{k-1} p^k (1-p)^{x-k}$$

- **p, k – distribution parameters**

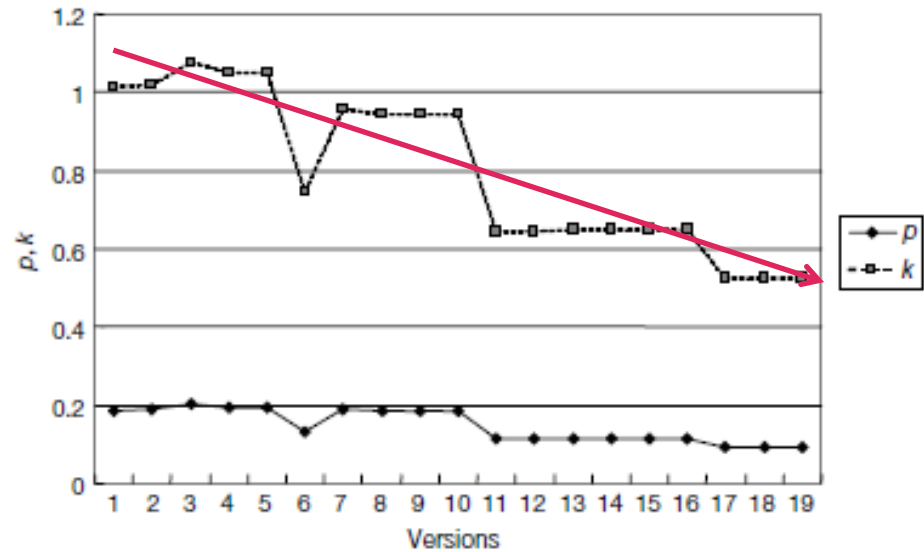
- $\binom{x-1}{k-1}$  - **binomial coefficient extended to the reals**



- **Increase – functionality enhancement**
- **Decrease – refactoring**

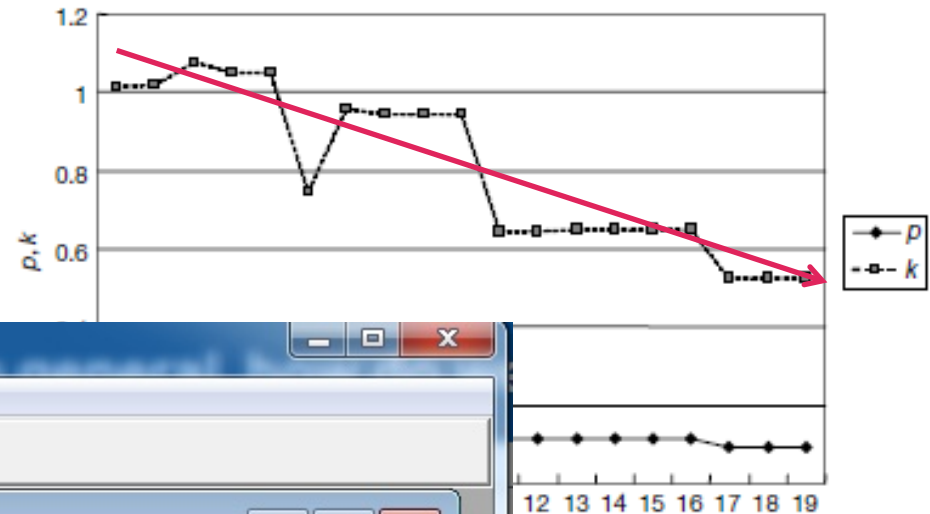
# In general, how do we study evolution?

- Visual inspection
  - Is this a real “trend” or just noise?



# In general, how do we study evolution?

- Time-series analysis
  - Simplest form: linear regression with *time*



```
> cor.test(data,time)

Pearson's product-moment correlation Linear trend
data: data and time
t = -7.4684, df = 16, p-value = 1.337e-06 Significant
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 -0.9552531 -0.7046643
sample estimates:
 cor
-0.8815246 Strong
```

More advanced techniques:

2DD23 - Time series analysis and forecasting

# Conclusions: Aggregation

- **Aggregation:**
  - **Metrics-independent**
    - **Applicable for any metrics to be aggregated**
    - **Traditional: mean, median...**
      - **“By no means”**
    - **Econometric: inequality indices**
  - **Metrics-dependent**
    - **Produce more precise results**
    - **BUT: need to be redone for any new metrics**
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# Metrics so far...

<b>Level</b>	<b>Metrics</b>
<b>Method</b>	<b>LOC, McCabe</b>
<b>Class</b>	<b>WMC, NOC, DIT, LCOM (and variants), LCC/TCC</b>
<b>Packages</b>	<b>Aggregation</b>
	<b>Direct metrics???</b>

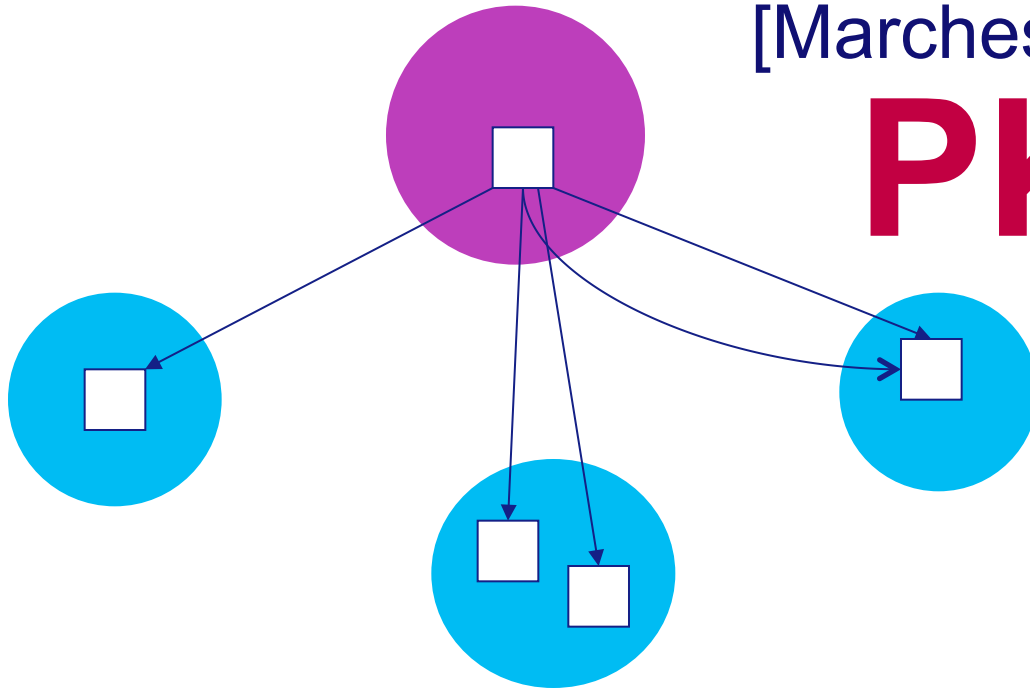
# Package metrics

- **Size:**
  - number of classes/interfaces
  - number of classes in the subpackages
- **Dependencies**
  - visualization
  - à la fan-in and fan-out
    - Marchesi's UML metrics
    - Martin's  $D_n$ : abstractness-instability balance or “the normalized distance from the main sequence”

# “Fan-out”

[Marchesi 1998]

[Martin 2000]



**PK<sub>1</sub> or R: 5**

**C<sub>e</sub>: 1**

[Martin 1994]

**3**

[JDepend]

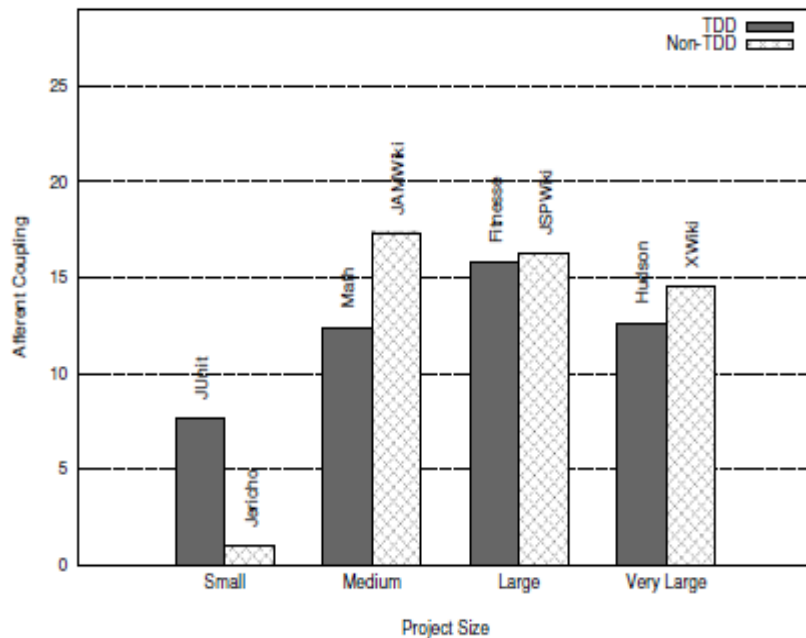
**4**

[Martin 2000]



# Fan-in

- “Fan-in” similarly to the “Fan-out”
  - Afferent coupling (Martin)
  - $PK_2$  (Marchesi)



- Dark: TDD, light: no-TDD
- Test-driven development positively affects  $C_a$ 
  - The lower  $C_a$  - the better.
- Exception: JUnit vs. Jerico
  - But Jericho is extremely small (2 packages)

[Hilton 2009]

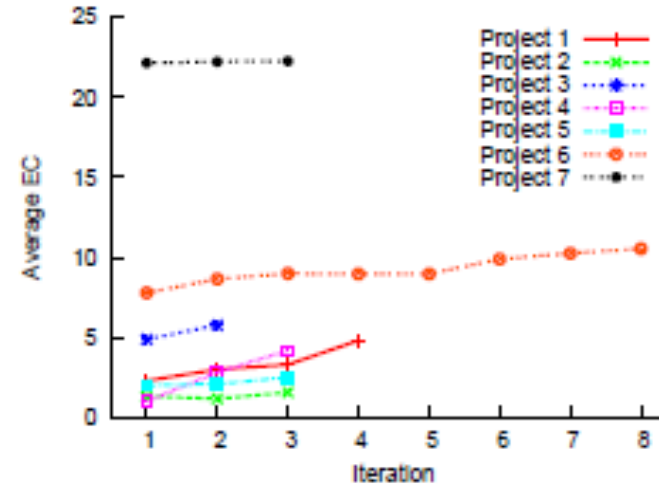
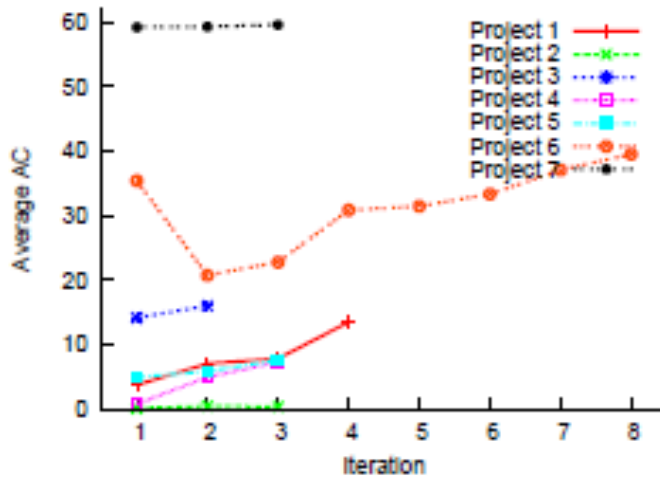
# More fan-in and fan-out

- “Fan-in” similarly to the “Fan-out”
  - Afferent coupling (Martin)
  - PK<sub>2</sub> (Marchesi)

<b>SAP (Herzig)</b>	<b>Correlation post-release defects</b>
<b>Afferent</b>	<b>0.091</b>
<b>Efferent [Martin 2000]</b>	<b>0.157</b>
<b>Class-in</b>	<b>0.084</b>
<b>Class-out</b>	<b>0.086</b>
<b>Fan-in</b>	<b>0.287</b>
<b>Fan-out</b>	<b>0.148</b>

<b>Marchesi</b>	<b>Man-months</b>	<b>#Pack</b>	<b>avg(PK<sub>1</sub>)</b>
<b>Railway simulator</b>	<b>13</b>	<b>6</b>	<b>8.7</b>
<b>Warehouse management</b>	<b>7</b>	<b>5</b>	<b>5.0</b>
<b>CASE tool</b>	<b>13</b>	<b>5</b>	<b>8.1</b>

# Evolution of afferent and efferent coupling



**Sato,  
Goldman,  
Kon 2007**

- Almost all systems show an increasing trend (Lehman's growing complexity)
- Project 7 (workflow system) is almost stable but very high!
  - Outsourced development
  - No automated tests
  - Severe maintainability problems

# Package metrics: Stability

*Stability is related to the amount of work required to make a change [Martin, 2000].*

- **Stable** packages
  - Do not depend upon classes outside
  - Many dependents
  - Should be extensible via inheritance (*abstract*)
- **Instable** packages
  - Depend upon many classes outside
  - No dependents
  - Should not be extensible via inheritance (*concrete*)

# What does balance mean?

A good real-life package must be **instable** enough in order to be easily modified

It must be **generic** enough to be adaptable to evolving requirements, either without or with only minimal modifications

Hence: contradictory criteria

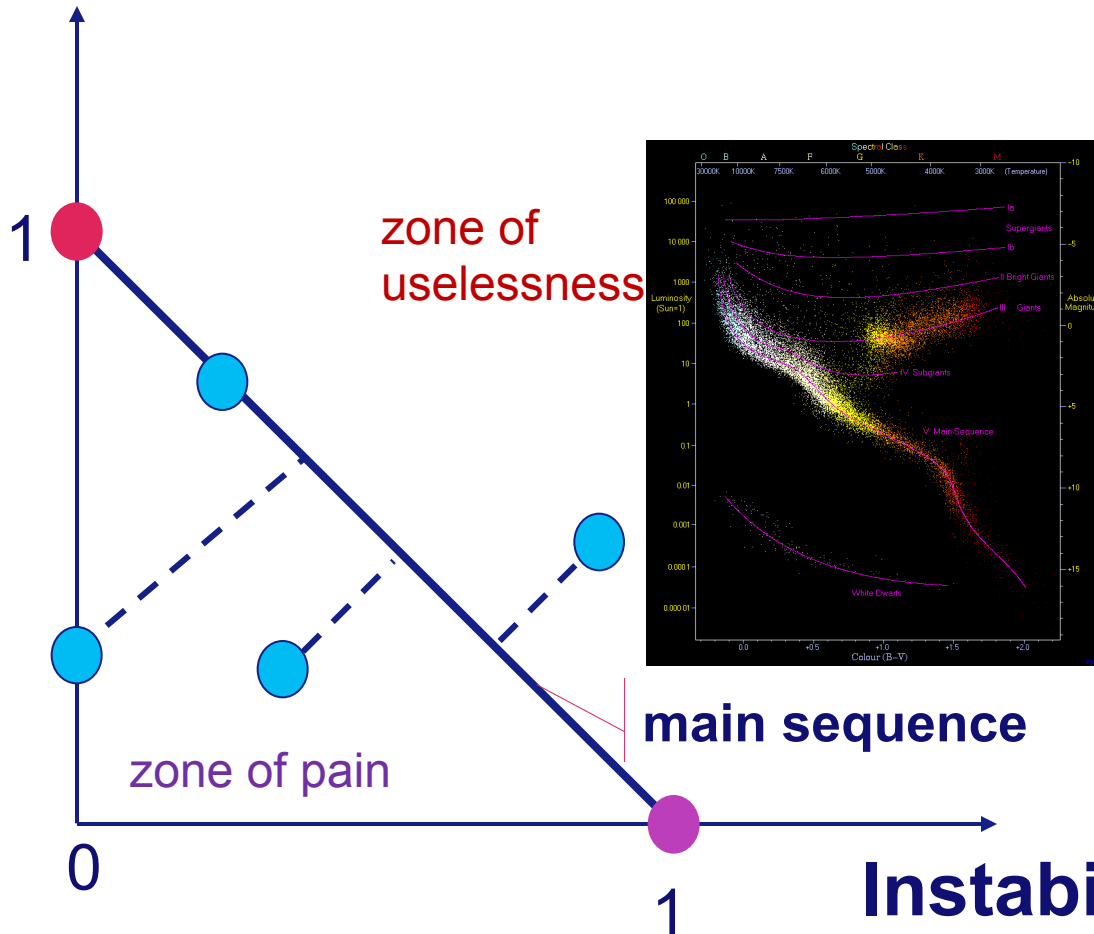
# $D_n$ – Distance from the main sequence

**Abstractness =**  
 $\#AbstrClasses/\#Classes$

$D_n =$

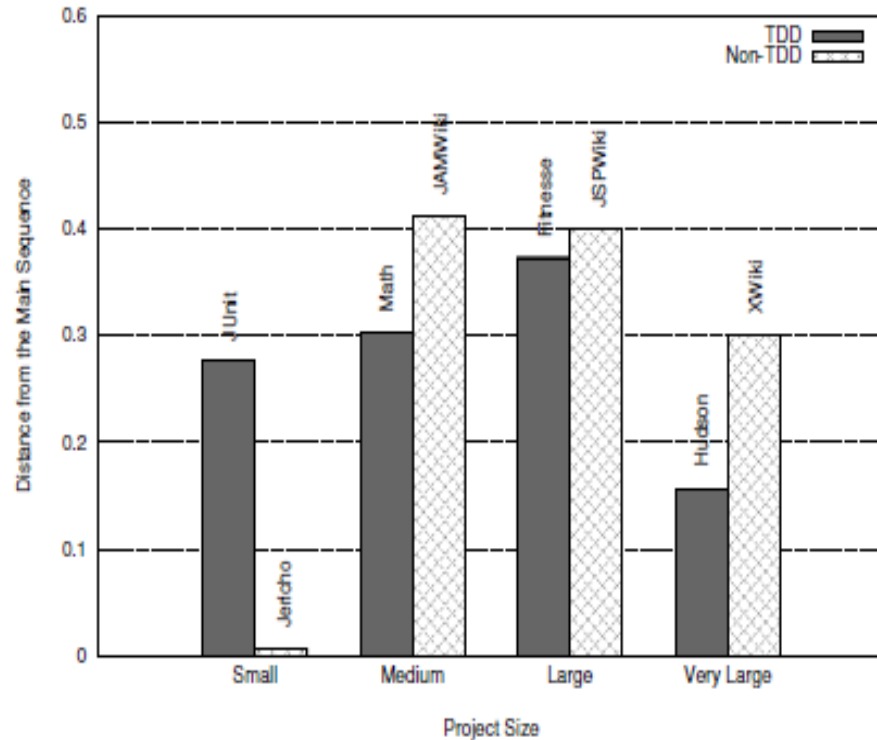
**| Abstractness +  
Instability – 1 |**

[R.Martin 1994]



**Instability =**  
 $C_e/(C_e+C_a)$

# Normalized distance from the main sequence

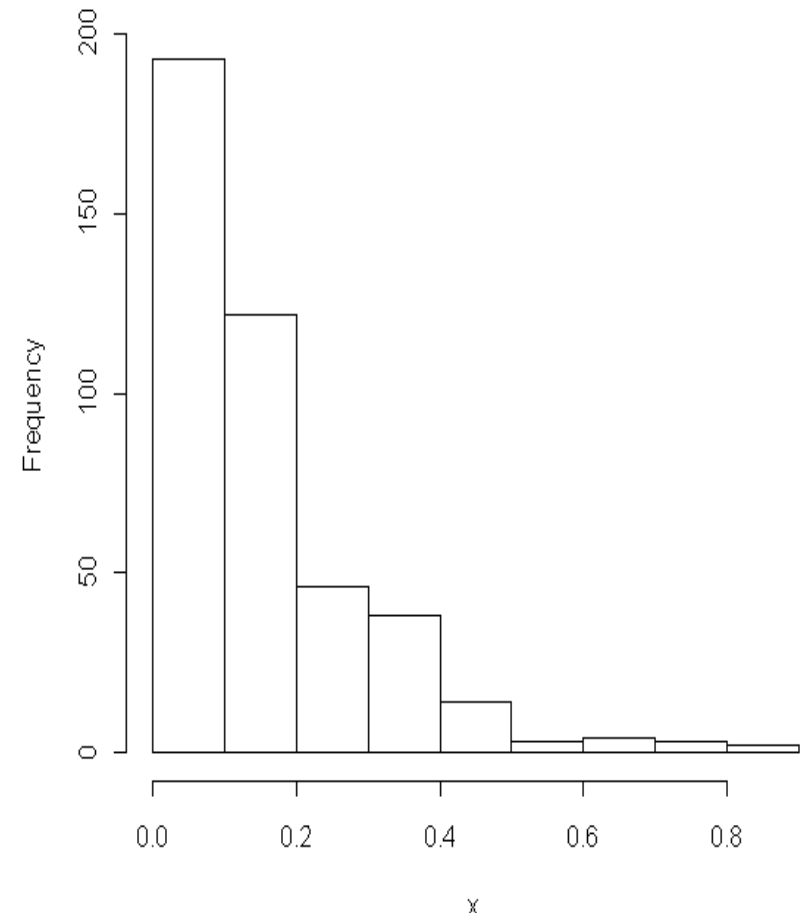


- Dark: TDD, light: no-TDD
- Test-driven development positively affects  $D_n$ 
  - The lower  $D_n$  - the better.
- The same exception (Jericho vs. JUnit)

[Hilton 2009]

## “Exponential” distribution

For all benchmark systems studied, here Vuze 4.0.0.4



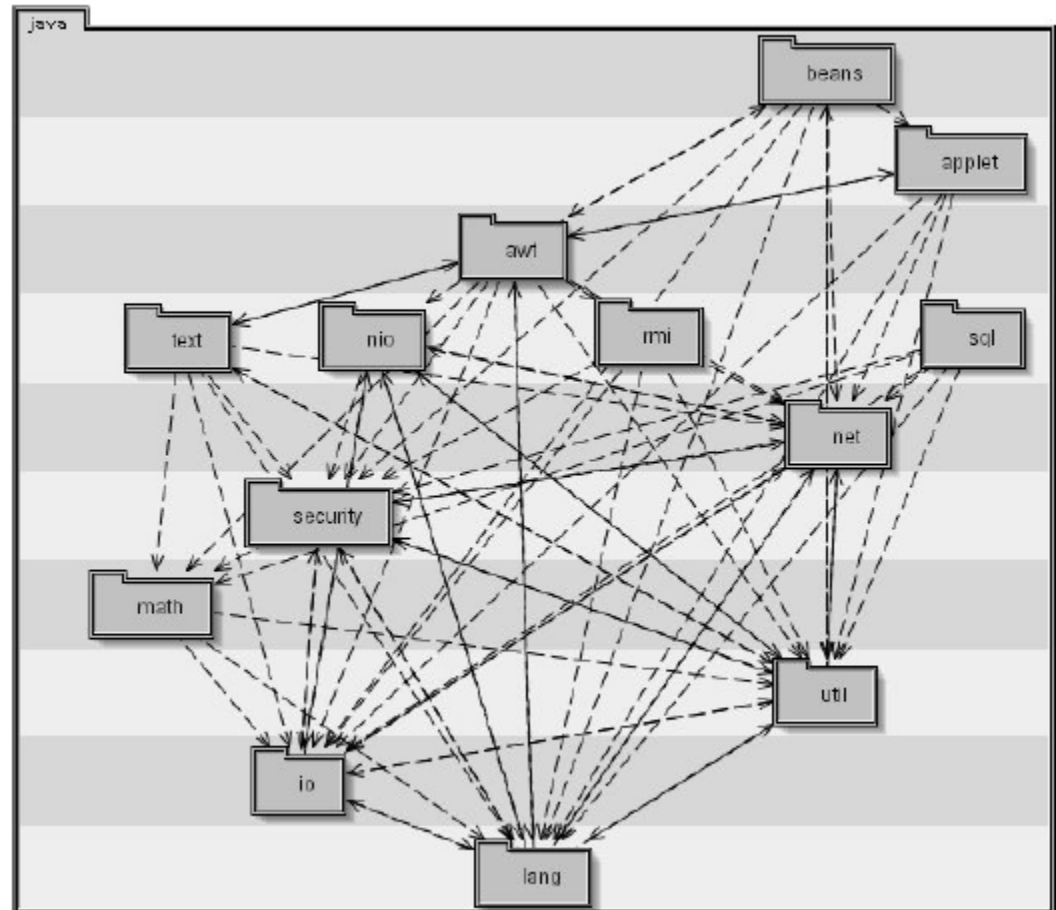


# PASTA [Hautus 2002]

- **PASTA – Package structure analysis tool**
- **Metrics**
  - **Similarly “fan-in/fan-out”**: based on dependencies between packages
  - **Go beyond calculating numbers of dependencies**
- **Focus on dependencies between the subpackages**
- **Some dependencies are worse than others**
  - **What are the “bad dependencies”?**
  - **Cyclic dependencies, layering violations**

# PASTA [Hautus]

- Idea: remove bad (cycle-causing) dependencies
- **Weight** – number of references from one subpackage to another one.
- Dependencies to be removed are such that
  - The result is acyclic
  - The total weight of the dependencies removed is minimal
- Minimal effort required to resolve all the cycles



**Upwards dependencies should be removed**

# From dependencies to metrics

- **PASTA(P) = Total weight of the dependencies to be removed / total weight of the dependencies**
- **No empirical validation of the metrics**
- **No studies of the metrics evolution**

Package	PASTA Metric
junit	0%
org.apache.batik	0%
org.apache.tools.ant	1%
java	5%
org.apache.jmeter	6%
javax.swing	10%
org.jboss	11%
org.gjt.sp.jedit	18%
java.awt	20%

# One metric is good, more metrics are better (?)

- **Recall...**

$$MI_1 = 171 - 5.2 \ln(V) - 0.23V(g) - 16.2 \ln(LOC)$$

**Halstead                  McCabe                  LOC**

- **[Kaur, Singh 2011] propose an adaptation...**

$$MIP = 171 - 5.2CC - 0.23 \ln(S) - 16.2 \ln(NC)$$

**Related to PK<sub>1</sub> and instability          Related to NOC and NOM          Related to nesting, strongly connected components, abstractness and PK<sub>2</sub>**

# Summary: package metrics

- **Size: number of classes**
- **Dependencies à la fan-in and fan-out**
  - **Marchesi's UML metrics**
  - **Martin's  $D_n$ : abstractness-instability balance or “the normalized distance from the main sequence”**
  - **PASTA**
- **Aggregations of class metrics: reminder**
  - **Metrics independent: average, sum, Gini/Theil coefficients**
  - **Metrics dependent: Distribution fitting**

# Measuring change: Churn metrics

- Why? Past evolution to predict future evolution
- Code Churn [Lehman, Belady 1985]:
  - Amount of code change taking place within a software unit over time
- Code Churn metrics [Nagappan, Bell 2005]:

## Absolute:

Churned LOC, Deleted LOC,  
File Count, Weeks of Churn,  
Churn Count, Files Churned

## Relative:

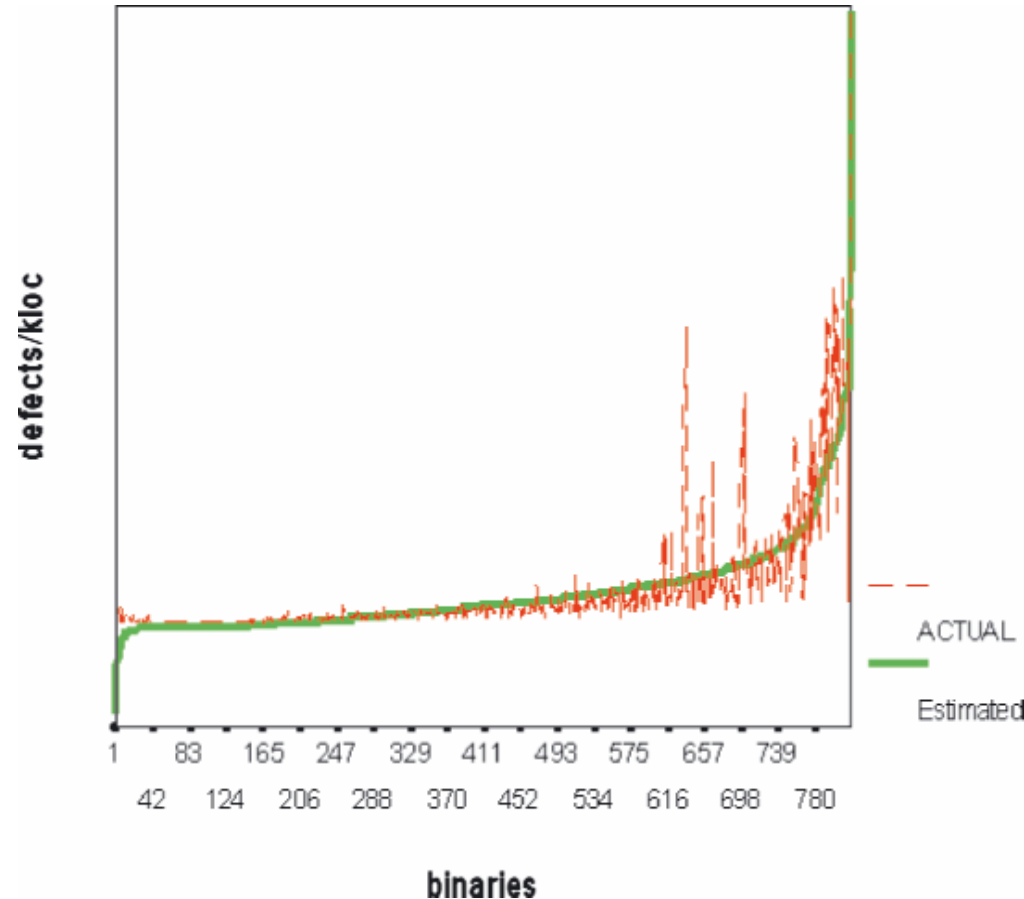
M1: Churned LOC / Total LOC  
M2: Deleted LOC / Total LOC  
M3: Files churned / File count  
M4: Churn count / Files churned  
M5: Weeks of churn / File count  
M6: Lines worked on / Weeks of churn  
M7: Churned LOC / Deleted LOC  
M8: Lines worked on / Churn count

# Case Study: Windows Server 2003

- **Analyze Code Churn between WS2003 and WS2003-SP1 to predict defect density in WS2003-SP1**
  - 40 million LOC, 2000 binaries
  - Use absolute and relative churn measures
- **Conclusion 1: Absolute measures are no good**
  - $R^2 < 0.05$
- **Conclusion 2: Relative measures are good!**
  - An increase in relative code churn measures is accompanied by an increase in system defect density
  - $R^2 \approx 0.8$

# Case Study: Windows Server 2003

- **Construct a statistical model**
  - Training set: 2/3 of the Windows Set binaries
- **Check the quality of the prediction**
  - Test set: remaining binaries
- **Three models**
  - **Right: all relative churn metrics are taken into account**





# Open issues

- To predict bugs from history, but we need a history filled with bugs to do so
  - Ideally, we don't have such a history
- We would like to learn from previous projects:
  - Can we make predictions without history?
  - How can we leverage knowledge between projects?
  - Are there universal properties?
- Not just **code properties** but also properties of the entire **software process**

# Conclusions

- **Package metrics**
  - **Directly defined:  $D_n$ , Marchesi metrics**
  - **Results of aggregation**
- **Churn metrics**